1 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

Factorising numerator :

$10x^2 + 23x + 12 \equiv (5x+4)(2x+3)$ (1)

Factorising denominator :

$4x^2 - 9 \equiv (2x+3)(2x-3)$ (1)

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} \equiv \frac{(5x+4)(2x+3)}{(2x+3)(2x-3)}$$

$$= \frac{5x+4}{2x-3}$$ (1)

$\dfrac{5x+4}{2x-3}$

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

$8^{5y} = (2^3)^{5y}$

$= 2^{15y}$ (1)

$4^n = (2^2)^n$

$= 2^{2n}$

$2^{2y} \times 2^{3y+2} = \dfrac{2^{15y}}{2^{2n}}$

$2^{2y+3y+2} = 2^{15y-2n}$ (1)

$2y + 3y + 2 = 15y - 2n$

$5y + 2 = 15y - 2n$ (1)

$2n = 15y - 5y - 2$

$2n = 10y - 2$

$n = 5y - 1$ (1)

$n = 5y - 1$

(4)

(Total for Question 1 is 7 marks)

2 Express

$$\left(\frac{4}{2x-5}-\frac{3}{2x-3}\right)\div\frac{9x-4x^3}{6x^2-17x+5}$$

as a single fraction in its simplest form.

$$\frac{4(2x-3)-3(2x-5)}{(2x-5)(2x-3)}$$

$$=\frac{8x-12-6x+15}{(2x-5)(2x-3)} \quad (1)$$

$$=\frac{2x+3}{(2x-5)(2x-3)}\times\frac{6x^2-17x+5}{9x-4x^3} \quad (1)$$

$$=\frac{2x+3}{(2x-5)(2x-3)}\times\frac{(3x-1)(2x-5)}{x(-2x+3)(2x+3)} \quad (1)$$

$$=\frac{3x-1}{x(2x-3)(-2x+3)} \quad (1)$$

$$\frac{3x-1}{x(2x-3)(-2x+3)}$$

(Total for Question 2 is 4 marks)

3 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5} \quad (1)$$

$$= \frac{5}{\frac{45}{5a-2} + \frac{5(5a-2)}{5a-2}}$$

$$= \frac{5}{\frac{45 + 25a - 10}{5a-2}} \quad (1)$$

$$= \frac{5(5a-2)}{35 + 25a}$$

$$= \frac{5(5a-2)}{5(7+5a)} \quad (1)$$

$$x = \frac{5a-2}{7+5a} \quad (1)$$

$x = \dfrac{5a-2}{7+5a}$

(Total for Question 3 is 4 marks)

4 (b) Write $\frac{2x+1}{4} + \frac{x-2}{3}$ as a single fraction in its simplest form.

Rationalise the denominator :

$\frac{3(2x+1) + 4(x-2)}{4(3)}$ (1)

$= \frac{6x+3+4x-8}{12}$ (1)

$= \frac{10x-5}{12}$ (1)

$\frac{10x-5}{12}$

(3)

(Total for Question 4 is 3 marks)

5 Solve the equation

$$\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$$

Show clear algebraic working.

$$\frac{5(x^2+2x) + 3(x+2)}{(x+2)(x^2+2x)} = 2 \quad (1)$$

$$5(x^2+2x) + 3(x+2) = 2(x+2)(x^2+2x) \quad (1)$$

$$5x^2+10x+3x+6 = 2(x^3+2x^2+2x^2+4x)$$

$$5x^2+13x+6 = 2(x^3+4x^2+4x)$$

$$5x^2+13x+6 = 2x^3+8x^2+8x$$

$$2x^3+8x^2-5x^2+8x-13x-6 = 0$$

$$2x^3+3x^2-5x-6 = 0 \quad (1)$$

$$(x+1)(2x-3)(x+2) = 0 \quad (1)$$

$$x = -1, 1.5, -2$$

Since $x+2 \neq 0$, x is equal to -1 and 1.5 (1)

ALTERNATIVE METHOD:

$$\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$$

$$\frac{5}{x+2} + \frac{3}{x(x+2x)} = 2$$

$$\frac{5x+3}{x^2+2x} = 2$$

$$5x+3 = 2(x^2+2x)$$

$$5x+3 = 2x^2+4x$$

$$2x^2-x-3 = 0$$

$$(2x-3)(x+1) = 0$$

$$x = -1 \text{ and } 1.5$$

-1 and 1.5

(Total for Question 5 is 5 marks)

6 (a) Express $\frac{4}{x-2} - \frac{3}{x+1}$ as a single fraction.

Give your answer in its simplest form.

$= \frac{4}{x-2} - \frac{3}{x+1}$

$= \frac{4(x+1) - 3(x-2)}{(x-2)(x+1)}$ (1)

$= \frac{4x+4-3x+6}{(x-2)(x+1)}$ (1)

$= \frac{x+10}{(x-2)(x+1)}$ (1)

$\frac{x+10}{(x-2)(x+1)}$

(3)

(Total for Question 6 is 3 marks)

7 (a) Solve $\frac{9a-7}{5} - \frac{3a-7}{4} = 4.55$

Show clear algebraic working.

$$\frac{4(9a-7) - 5(3a-7)}{5 \times 4} = 4.55$$

$$\frac{36a - 28 - 15a + 35}{20} = 4.55$$

$$21a + 7 = 4.55 \times 20$$

$$21a + 7 = 91 \quad (1)$$

$$21a = 91 - 7$$

$$21a = 84 \quad (1)$$

$$a = 4 \quad (1)$$

$a =$ 4

(3)

(b) Make c the subject of the formula $p = \sqrt{\frac{ac+8}{3+c}}$

$$p = \sqrt{\frac{ac+8}{3+c}}$$

$$p^2 = \frac{ac+8}{3+c} \quad (1)$$

$$p^2(3+c) = ac + 8$$

$$3p^2 + p^2c = ac + 8 \quad (1)$$

$$3p^2 - 8 = ac - p^2c \quad (1)$$

$$3p^2 - 8 = c(a - p^2)$$

$$c = \frac{3p^2 - 8}{a - p^2} \quad (1)$$

$\frac{3p^2-8}{a-p^2}$

(4)

(Total for Question 7 is 7 marks)

8 Write $\dfrac{25x^2 - 64}{5x^2 - 13x - 6} \times \dfrac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

as a single fraction in its simplest form.
Show clear algebraic working.

$= \dfrac{25x^2 - 64}{5x^2 - 13x - 6} \times \dfrac{x^2 - 8x + 15}{5x + 8} - (x - 7)$

$= \dfrac{(5x + 8)(5x - 8)}{(5x + 2)(x - 3)} \times \dfrac{(x - 5)(x - 3)}{(5x + 8)} - (x - 7)$

$= \dfrac{(5x - 8)(x - 5)}{(5x + 2)} - (x - 7)$ (2)

$= \dfrac{(5x - 8)(x - 5) - (x - 7)(5x + 2)}{(5x + 2)}$

$= \dfrac{5x^2 - 25x - 8x + 40 - (5x^2 + 2x - 35x - 14)}{5x + 2}$ (1)

$= \dfrac{5x^2 - 25x - 8x + 40 - (5x^2 - 33x - 14)}{5x + 2}$

$= \dfrac{5x^2 - 33x + 40 - 5x^2 + 33x + 14}{5x + 2}$

$= \dfrac{54}{5x + 2}$ (1)

$\frac{54}{5x+2}$

.....

(Total for Question 8 is 4 marks)

9 Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

Simplifying terms in bracket into single fraction:

$\frac{20}{x^2-36} - \frac{2}{x-6}\,\frac{(x+6)}{(x+6)}$

$= \frac{20}{x^2-36} - \frac{2(x+6)}{x^2-36}$ (1)

$= \frac{20-2(x+6)}{x^2-36}$

$= \frac{8-2x}{x^2-36} = \frac{2(4-x)}{x^2-36}$

Multiply with the remaining fraction:

$\frac{2(4-x)}{x^2-36} \times \frac{1}{4-x}$ (1)

$= \frac{2}{x^2-36}$ (1)

$\frac{2}{x^2-36}$

(Total for Question 9 is 3 marks)

10 Write

$$\frac{4x^2 - 17x - 15}{2x - 1} \times \frac{2x^2 - 7x + 3}{x^2 - 25} + (29 - 4x)$$

as a single fraction in its simplest form.

$\frac{(4x+3)(x-5)}{2x-1} \times \frac{(2x-1)(x-3)}{(x+5)(x-5)}$ (1) $= \frac{(4x+3)(x-3)}{x+5}$

$= \frac{4x^2 - 12x + 3x - 9}{x+5}$

$= \frac{4x^2 - 9x - 9}{x+5}$ (1)

$\frac{4x^2 - 9x - 9 + (x+5)(29-4x)}{x+5}$ (1) $= \frac{4x^2 - 9x - 9 + 29x - 4x^2 + 145 - 20x}{x+5}$

$= \frac{-9 + 145}{x+5}$

$= \frac{136}{x+5}$ (1)

$\frac{136}{x+5}$

(Total for Question 10 is 4 marks)